CLEAR AIR TURBULENCE FREQUENCY AS A FUNCTION OF WIND SHEAR AND DEFORMATION [1]

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ABSTRACT

The probability is determined that an aircraft will encounter moderate or severe high-level turbulence during a 100-mi. flight segment when particular values of certain meteorological quantities exist in that locality. The turbulence data used are pilot reports collected by the U.S. Weather Bureau Clear Air Turbulence Project for March 12–24, 1962 and February 4–9, 1963. The meteorological quantities which were computed from standard data include vertical vector wind shear, vertical wind direction shear, temperature lapse rate, horizontal wind shear, vorticity, and resultant deformation. A correlation of 0.45 was found between turbulence frequency and the product of vertical vector wind shear and deformation. This value is the highest correlation found so far with data of this type. The product of wind shear and deformation is an important factor in the development of fronts. One might expect that a tendency for frontogenesis would correlate better with turbulence than would frontolysis. The data however, indicate that both processes are equally important. Temperature lapse rate appeared to have little influence on the turbulence frequency except for a few occasions when conditions were nearly dry adiabatic. The regression equations between turbulence frequency and meteorological quantities that have been derived appear useful in estimating the risk of encountering turbulence in a given locality. Such turbulence estimates can be made at a particular time from the concurrent upper-air data, or on a climatological basis from the climatology of the pertinent meteorological factors.

1. INTRODUCTION

Recent literature contains a number of articles concerned with the association of meteorological conditions in the free atmosphere and concurrent turbulence (Colson [2], Briggs and Roach [1], Kronebach [9], Endlich [5], and Reiter and Nania [12]). Conditions that have generally been related to the occurrence of turbulence are: small Richardson's number, large vertical and horizontal wind shear, and large spatial variations in the thermal structure of the atmosphere. Vertical wind direction shear (the product of wind speed and the vertical change of wind direction) was found by Endlich and McLean [6] to have a relatively high correlation with turbulence, and appeared somewhat superior to other meteorological quantities analyzed in their data sample. Colson and Panofsky [4] found that an index based on both the square of the vertical wind shear and Richardson's number provided a better discrimination between regions with and without turbulence than did either wind shear or the Richardson number alone. The circumstances associated with turbulence support the hypothesis of Kuettner [10] that unstable shear-gravity waves occurring in certain portions of jet streams are a prime source of clear air turbulence. A detailed analysis by Holmboe [8] has shown that small values of Richardson's number favor the amplification of waves smaller than a few kilometers in length. These same meteorological conditions favor the growth or continuation of turbulent eddies, so that both wave and eddy considerations are consistent.

The practical problem of turbulence analysis can be considered as one of identifying these unstable mesoscale regions from available data. Because of the inherent uncertainties associated with determining the atmospheric mesostructure from routine observations, the best techniques that can be devised will most likely describe turbulence occurrence in probabilistic terms. In searching for better methods of determining turbulence probability, the writers have made use of the large number of pilot reports of turbulence and non-turbulence over the United States compiled by Colson [2], [3]. Concurrent upper-air (rawinsonde) data for stations throughout the United States were analyzed by computer at grid points located at the centers of regions 2.5° latitude by 2.5° longitude and 50 mb. thick. These regions were also used for compiling the number of turbulent and nonturbulent flight traverses. Analyses were made of winds, vertical wind shear, and various derived horizontal properties such as relative vorticity and resultant deformation. The 5-day period February 4–9, 1963 was used to derive relationships between meteorological quantities and frequency of turbulence occurrence. These relation-

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ships were then checked against a set of independent data for the period March 12–24, 1962.

2. METEOROLOGICAL QUANTITIES ANALYZED

Prior to making grid point analyses, a number of vertical derivatives were computed from individual rawinsonde reports and compared with nearby turbulence observations. These quantities included wind speed, wind speed shear, wind direction shear, wind vector shear, temperature lapse rate, Richardson's number, wind vector curvature, and Scorer's number. The better discriminators of turbulence were found to be the wind shear terms. Table 1 shows the dependence of turbulence frequency on wind vector shear ($\Delta \mathbf{V}/\Delta z$) and temperature lapse rate ($-\Delta T/\Delta z$) in the upper troposphere (20,000 to 40,000 ft.). The number in parenthesis in each category shows the number of cases used in the analysis. The increase of turbulence frequency with shear is clear for each category of lapse rate (columns in table 1); however, for each shear category (rows in table 1) the frequencies do not increase as lapse rates become more unstable. The overall tendency appears to show a slight increase in turbulence frequencies with stable lapse rates and also for approximately dry adiabatic lapse rates. In the latter case, on the few occasions (29) when the shear was greater than 0.008 sec.$^{-1}$, a high proportion (28 percent) reported moderate or severe turbulence. The normal interpretation of lapse rate (as in Richardson's number) seems to be applicable only to these few cases when lapse rate was greater than 9°K. km.$^{-1}$ Therefore, this analysis casts doubt on the validity of using lapse rate as a variable in turbulence analysis.

The relative importance of vertical wind speed shear ($\Delta V/\Delta z$) and vertical direction shear ($V\Delta\alpha/\Delta z$) was also tested in a similar manner. These two orthogonal components of the vector shear were computed separately and their relationship to the frequency of moderate or severe turbulence is shown in table 2. The relationship of each to turbulence is quite similar, so that the vector shear (which includes both terms) appears to be preferable to either term used alone.

TABLE 1.—*Percent of flight segments reporting moderate or severe turbulence in the upper troposphere as a function of vector wind shear and temperature lapse rate over 50-mb. layers (data for February 4–9, 1963). The number of reports for each entry is given in parentheses*

Wind Shear sec.$^{-1}$	Lapse Rate °K. km.$^{-1}$					
	<4 Stable	4–7	7–8	8–9	>9 Unstable	Total
0–0.004	4.3 (304)	3.8 (629)	3.4 (1865)	2.8 (4389)	4.7 (446)	3.2 (7633)
0.004–0.008	11.2 (267)	5.3 (678)	6.3 (1648)	5.5 (2262)	10.3 (261)	6.3 (5116)
>0.008	12.4 (371)	11.6 (666)	12.3 (608)	13.1 (512)	27.6 (29)	12.5 (2186)
Total	9.4 (942)	6.9 (1973)	5.9 (4121)	4.4 (7163)	7.6 (736)	5.6 (14935)

In the grid point analyses, the wind speed V, the vertical wind vector shear ($\Delta\mathbf{V}/\Delta z$) and the wind direction shear ($V\Delta\alpha/\Delta z$)—all for 50-mb. layers—were computed. The technique used to transform station data to grid points (region centers) has been described by Endlich and Mancuso [7]. Horizontal flow properties analyzed at grid points included the horizontal wind vector shears both parallel to the flow direction ($\Delta\mathbf{V}/\Delta s$) and normal to the flow ($\Delta\mathbf{V}/\Delta n$) computed from formulas given by Saucier [13]. Relative vorticity (ζ) and resultant deformation (Def) were computed from the finite difference formulas,

$$\zeta=\Delta v/\Delta x-\Delta u/\Delta y \quad (1)$$

$$\text{Def}=[(\Delta u/\Delta x-\Delta v/\Delta y)^2+(\Delta v/\Delta x+\Delta u/\Delta y)^2]^{1/2} \quad (2)$$

where u is the wind component along the x direction (east-west) and v is the wind component along the y direction (north-south). Deformation may be described as a kinematic property of the flow that tends to transform an original circle of fluid into an elongated elliptical shape. The long axis of the ellipse is called the axis of dilatation.

Since deformation has not previously been considered to be a factor in clear air turbulence, it is interesting to consider possible reasons for its importance. One might suspect that large deformation is favorable for the generation of mesoscale waves that can degenerate into turbulence. At present, this concept lacks theoretical support. Also, it is known that deformation is an important factor in producing or destroying horizontal gradients of properties such as temperature (Petterssen [11]). When the axis of dilatation of the deformation field is oriented within 45° of the horizontal isotherms, the pre-existing temperature gradient is increased; otherwise it is weakened. The rate of change of temperature gradient (accumulation of isotherms) on a constant pressure surface caused by horizontal deformation is given by

$$A_T=d(\partial T/\partial n)/dt=0.5(\partial T/\partial n)(\text{Def})\cos 2\beta \quad (3)$$

TABLE 2.—*Percent of flight segments reporting moderate or severe turbulence as a function of vertical speed shear $\Delta V/\Delta z$ and vertical direction shear $V\Delta\alpha/\Delta z$ over 50-mb. layers (data for February 4–9, 1963). The number of reports for each entry is given in parentheses*

Direction Shear, sec.$^{-1}$	Speed Shear, sec.$^{-1}$					
	0–0.002	0.002–0.004	0.004–0.006	0.006–0.008	>0.008	Total
0–0.002	2.7 (3269)	3.7 (2120)	5.4 (1181)	8.6 (443)	9.3 (398)	4.1 (7411)
0.002–0.004	3.5 (1956)	5.0 (1130)	4.5 (663)	10.5 (324)	6.5 (231)	4.7 (4304)
0.004–0.006	5.4 (661)	5.7 (279)	9.4 (265)	10.1 (129)	14.5 (179)	7.7 (1513)
0.006–0.008	5.7 (331)	9.9 (121)	7.1 (126)	28.6 (42)	23.5 (166)	11.6 (786)
>0.008	12.8 (296)	13.0 (246)	14.4 (153)	17.9 (84)	13.3 (143)	13.7 (922)
Total	3.8 (6513)	5.0 (3896)	6.3 (2388)	11.0 (1022)	12.2 (1117)	5.6 (14936)

where n is the horizontal axis normal to the isotherms, and β is the angle between the isotherms and the axis of dilatation. Because the temperature gradient may be related to the vertical wind shear by the thermal wind relationship, (3) may be rewritten as

$$A_T = 0.5(fT/g)(\Delta\mathbf{V}/\Delta z)(\mathrm{Def}) \cos 2\beta \tag{4}$$

where f is the Coriolis parameter and g is gravity. Both A_T and the product (Def)($\Delta\mathbf{V}/\Delta z$) were among the quantities analyzed at grid points.

3. RESULTS FOR GRID-POINT ANALYSES

The upper-air charts for February 4–9, 1963 show that a series of sharp troughs entered the northwestern coast of the United States and moved eastward with deepening occurring along the east coast. There were no pronounced jet streams and only a few wind speeds exceeded 50 m. sec.$^{-1}$ The regions of turbulence appeared to be generally associated with the troughs, and these contained larger than average values of quantities such as vertical wind shear and horizontal deformation.

The relationships between frequency of occurrence of turbulence and the objectively analyzed meteorological quantities were examined statistically by computing linear correlation coefficients and lines of regression. Only the regions having six or more flight traverses (within a 12-hr. interval) were used to compute turbulence frequencies. These values were then correlated with the mid-interval values of the meteorological quantities computed at the centers of the regions (grid points). Thus, the number of reports used in the correlation computation was restricted to 4977 (of a total of 8542 available), and from these reports it was possible to compute 513 grid point values of turbulence frequency. The results are summarized in table 3 for both the 350–300-mb. and the 300–250-mb. layers, and in table 4 for both layers combined.

In agreement with the findings of Colson [3] for this period, the regional values of relative vorticity in both layers (table 3) showed definite correlations (0.26 and 0.27) with the percent of turbulence reports; that is, high positive values of vorticity were associated with frequent reports of turbulence. Similar correlation coefficients (0.22 to 0.37) were obtained for the magnitude of vector wind shear along the direction of flow $\Delta\mathbf{V}/\Delta s$. The cross flow vector wind shear $\Delta\mathbf{V}/\Delta n$ had a small correlation (0.17) in the 350–300-mb. layer, but interestingly, none in the 300–250-mb. layer. This low value is not in agreement with a high correlation (0.8) found between turbulence (measured by a B-47) and the cross flow shear term $V\Delta\alpha/\Delta n$ measured on the mesoscale. This difference may be due to the low resolving power of standard measurements in representing this term. Wind speed, as had been expected, showed very little correlation with turbulence. The horizontal deformational field in comparison to the above-mentioned horizontal properties gave relatively high correlations (0.37 and 0.41). The correlations (0.26 and 0.18) obtained for the vertical wind vector shear were somewhat smaller than those for the wind direction shear (0.31 and 0.27). Although none of the correlations is very large, it is believed that they indicate the relative value of the different quantities.

The highest correlations obtained (0.48 and 0.43) were for the magnitude of the product of vertical wind vector shear and deformation. The absolute value of A_T had only slightly smaller correlations (0.44 and 0.39); however, when the sign of A_T was retained there was no correlation with turbulence frequency.

To test these results further, a stepwise regression computer routine was used on the 350–300-mb. data to determine the optimum linear and logarithmic (product) combinations for this layer. The best single combination selected by the routine was the product of vector (or directional) wind shear and deformation, while inclusion of any of the other quantities that had been analyzed added rather insignficant improvement to the relationship with turbulence.

Also listed in table 4 are the two coefficients for the lines of regression which linearly relate percent of turbulence occurrence to values of vertical wind shear, horizontal deformation, and their product. The regression lines for the latter two quantities are plotted (solid lines) in figure 1 together with presentations (dash-dot lines) of the turbulence frequencies obtained for aircraft reports grouped within class intervals of the variate. The percent of aircraft reports (out of a total of 8542 cases) within each of the class intervals is also shown (dashed lines) for completeness. The regression line for the product provides an excellent fit to the data. In the case of deformation however, a better fit to the actual turbu-

Table 3.—*Coefficients of linear correlation between the frequency of moderate or severe turbulence within a region and concurrent grid-point values (magnitudes) of various meteorological quantities [based on 2782 (350–300 mb.) and 2195 (300–250 mb.) reports for February 4–9, 1963]*

Layer (mb.)	V	$\Delta\mathbf{V}/\Delta n$	$\Delta\mathbf{V}/\Delta s$	ζ	Def	$\Delta\mathbf{V}/\Delta z$	$V\Delta\alpha/\Delta z$	Def $\cdot\,\Delta\mathbf{V}/\Delta z$	$\lvert A_T\rvert$
350–300	0.03	0.17	0.22	0.26	0.37	0.26	0.31	0.48	0.44
300–250	0.08	0.01	0.37	0.27	0.41	0.18	0.27	0.43	0.39

Table 4.—*Linear relationships between and percent of turbulence within a region and the concurrent values of selected meteorological quantities [based on 4977 reports (combined layers) for February 4–9, 1963]*

Quantities	Correlation Coefficient	Line of Regression $P=a+bQ$	
		a	b
$\Delta\mathbf{V}/\Delta z$ (10^{-3} sec.$^{-1}$)	0.22	0.4	1.2
Def (10^{-5} sec.$^{-1}$)	0.39	−0.7	2.1
Def$\cdot(\Delta\mathbf{V}/\Delta z)$ (10^{-7} sec.$^{-2}$)	0.45	−0.7	4.6

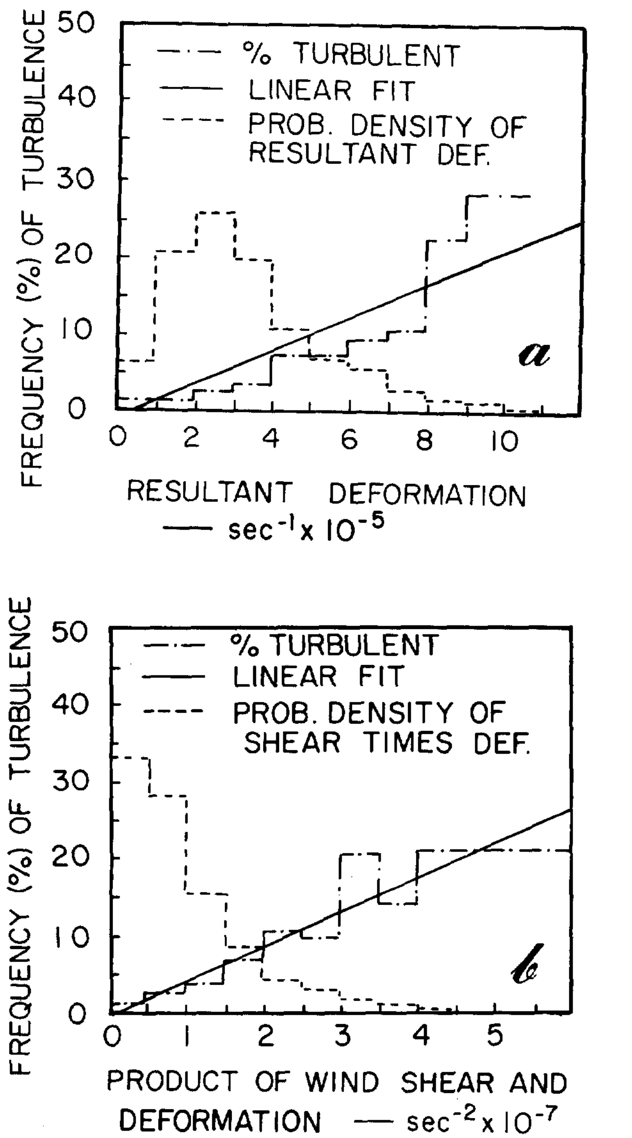

Figure 1.—Frequency (percent) of moderate or severe turbulence in 100-mi. flight segments (data for February 4–9, 1963) (*a*) as a function of magnitude of resultant deformation, and (*b*) as a function of product of vertical vector shear and deformation.

lence frequency would have been provided by a nonlinear curve; for example, a curve varying proportionally to Def^2.

Turbulence reports for March 12–24, 1962 were used as a check on the significance of the product of shear and deformation. During this period, the upper-air synoptic charts were characterized by strong jet streams and rapidly changing patterns. Because of the relatively strong winds, a number of the radiosonde reports lacked upper-wind data at the levels of analysis. This produced some difficulty because the regions of turbulence tended to be associated with the jet stream in contrast to the February period when they appeared to concentrate in the troughs. Therefore, the analyses for the March period were restricted to the 350–300-mb. layer (and 1698 aircraft reports) for which significantly fewer winds were

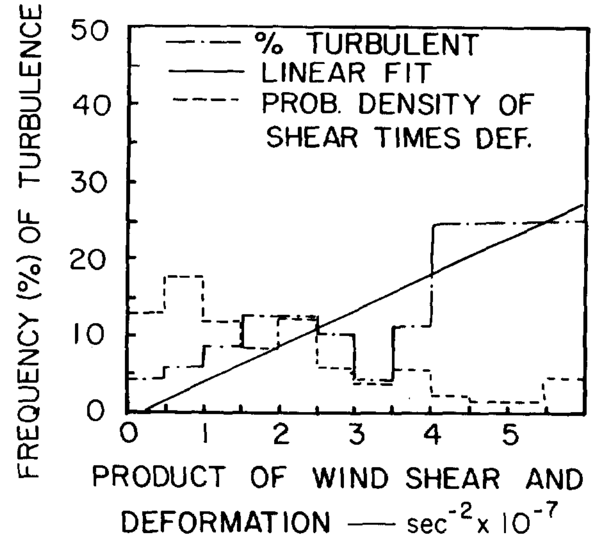

Figure 2.—Frequency (percent) of moderate or severe turbulence in 100-mi. flight segments as a function of magnitude of product of vertical vector shear and deformation (data for 350–300-mb. layer, March 12–24, 1962).

missing than for the 300–250-mb. layer. The March data results are plotted (dash-dot lines) in figure 2 and overall are in good agreement with the previously derived regression line (solid line). Thus these results further substantiate the significance of the product of shear and deformation and the validity of using it as a turbulence criterion.

4. DISCUSSION

In correlating turbulence frequency with meteorological quantities, one is limited by the resolution and accuracy of standard data, as noted by many writers. For example, certain quantities such as geostrophic departure and divergence might have mesoscale patterns that correlate with turbulence. Such small terms, however, cannot now be computed very reliably and in practice do not correlate well with turbulence. However, our experience has been that vertical shear, temperature lapse rate, vorticity, horizontal shear, and deformation can be computed with considerable confidence over areas of dense data such as the United States. Thus the correlation of the product of shear and deformation with turbulence is based on reasonable computations and is not regarded as an accidental result. Further testing, which is presently being performed on a more recent reporting period (Dec. 9–14, 1964), is also showing that the product has a relationship with turbulence almost identical to that given by the regression equation of table 4.

At this time we do not have a completely satisfactory explanation for the physical connection between turbulence and the product of vertical wind shear and deformation. In part, the utility of this quantity may be due to the fact that it reflects wind variations in three dimensions. As mentioned earlier, the relationship may be an indirect one associated with the creation and destruction of gradients of wind or temperature.

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